$\exists x,y \in \mathbb{Q}$ mit x

$\to \underline{x} \le 5+y$ ✓

$x-y>7 \to \underline{x} > 7+y$

$x+y<9 \to \underline{x} < 9-y$

$$7+y \le x \le 5+y$$

* $7+y < 5+y \rightsquigarrow 7 < 5$

$$7+y < x < 9-y$$

* $7+y < 9-y$

$y > 5 \qquad \{y \mapsto 6\}$

$t[y] < x < t[y]$

$\underbrace{(t[y])\{y \to 6\}}_{q_1 \in \mathbb{Q}} < x < \underbrace{(t[y])\{y \to 6\}}_{q_2 \in \mathbb{Q}}$

$3 \qquad x \to \frac{q_2 - q_1}{2} \in \mathbb{Q} \qquad 5$

LRA $\in$ P

LIA $\in$ NP

$\frac{5-3}{2} \to 4$

Propositional clause set N

$\cancel{P \vee C_i} \in N$ $\quad |N| = m$

$\cancel{\neg P \vee D_j} \in N$

Eliminate P

$|C_i \vee D_j| = O(m^2)$

$\leq m$

Variable Elimination

$\forall x \quad \neg P(x) \vee P(f(x))$

$P \notin C_i$ $\quad P \vee P \vee C_i'$

$P \notin D_i$ $\quad P \vee C_i'$

$\cancel{P \vee \neg P \vee C_i'}$

Φ in DNF

$\exists x \exists y \; \Phi$

$\exists x \; \Phi'$, DNF

no exp blow-up because DNF

$\exists^*$

$\leadsto$ quantifier alternation causes complexity

$\forall x \forall y \; \Phi$

$\forall x \neg \underbrace{\exists y \neg \Phi}$

$\forall x \neg \Phi'$

$\neg \exists x \cancel{\neg} (\neg \Phi')$

FOL

$\forall^{*}_{x_i} \Phi$ (Φ without functions)

decidable

$\exists^{*}_{x_i} \Phi$ "

$\forall x \exists y \forall x' \exists y'$ undecidable

$\exists x \, \forall y \, (3x < y)$ false

$\exists x \, \neg \exists y \, \neg (3x < y)$

$\exists x \, \neg \exists y \, [(3x \geq y) \vee (x \leq 6)]$

eliminate $\cancel{y}$ $3x \geq 0$

Elim $\exists$

$\Longrightarrow$ FM

$\emptyset$ True

$\exists x \, \neg \, (3 \geq 0)$ $y \to 0$

$x \to n$

$\exists x \, (3 < 0)$ false