max planck institut
informatik

**Universität des Saarlandes**
**FR Informatik**


**Christoph Weidenbach** **January 31, 2017**

**Tutorials for "Automated Reasoning"**
**Exercise sheet 11**

**Exercise 11.1:** (*4+4 P*)
Refute the following set $N$ of clauses

| | | | |
|---|---|---|---|
| (1) | $P(a,b) \vee P(b,a)$ | (2) | $\neg P(a,b) \vee P(f(b,b),b)$ |
| (3) | $\neg P(b,a) \vee Q(g(a))$ | (4) | $\neg Q(g(a)) \vee P(f(b,b),b)$ |
| (5) | $\neg P(f(b,b),b) \vee \neg P(f(b,b),b)$ | | |

both using KBO and LPO with ground superposition by only applying the inference rules Superposition Left and Factoring:

1. using KBO where all variables and signature symbols have weight 1 and $Q \succ P \succ f \succ g \succ b \succ a$,
2. using LPO with precedence $Q \succ P \succ f \succ g \succ b \succ a$.

**Exercise 11.2:** (*4+4 P*)
Consider again the above clause set. This time compute the model $N_{\mathcal{I}}$ both for KBO and LPO:

1. using KBO where all variables and signature symbols have weight 1 and $Q \succ P \succ f \succ g \succ b \succ a$. Compute $N_{\mathcal{I}}$, determine the minimal false clause, perform the respective ground superposition inference, add the result to $N$ yielding $N'$ and compute again $N'_{\mathcal{I}}$,
2. using LPO with precedence $Q \succ P \succ f \succ g \succ b \succ a$. Compute $N_{\mathcal{I}}$, determine the minimal false clause, perform the respective ground superposition inference, add the result to $N$ yielding $N'$ and compute again $N'_{\mathcal{I}}$.

**Exercise 11.3:** (*2+1 P*)
Let $N$ be a ground clause set. Consider the following graph $G$ out of $N$ where the nodes of the graph are the predicates from $N$ and the links are defined as follows: for all clauses $C \in N$ and all pairs of a negative literal $\neg P(t_1, \ldots, t_n) \in C$ and a positive literal $Q(s_1, \ldots, s_m) \in C$ add the directed link $P \mapsto Q$ to $G$. Now prove the following: if $G$ is cycle free, then $N$ can be finitely saturated by ordered ground superposition.

Does the reverse, if $N$ can be finitely saturated then $G$ is cycle free, hold as well?

Submit your solution in lecture hall E1.3, Room 001 during the lecture on February 07. Please write your name and the date/time of your tutorial group (Wed-Fabian, Wed-Tobias) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.