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**Tutorials for “Automated Reasoning”  
Exercise sheet 6**

**Exercise 6.1:** (8 P)

Show a superposition derivation of the empty clause (notation:  $\perp$ ) from this set of clauses:  $N = \{R \vee S, R \vee P \vee \neg Q, \neg Q \vee \neg S, \neg P \vee Q \vee \neg R, P \vee Q, \neg R \vee S, \neg P \vee \neg S\}$ . Ordering on atoms is:  $P \succ Q \succ R \succ S$ . Use only the rules Superposition Left, Factoring, and Condensation. Present the derivation due to the model building process, i.e., build the partial model, do the inference on the minimal false clause, update the partial model and continue this process until you derive  $\perp$ . You may use Condensation to get rid of duplicate literals.

**Exercise 6.2:** (3 P)

Let  $a : \rightarrow S$  and  $R \subseteq S \times T$ . Complete the sort information for  $g, f, P, a$  and variables  $x, y$  such that the following formula is well-sorted:  $\forall x, y. (R(x, g(x)) \rightarrow (f(g(x), a) \approx y \vee P(y) \vee R(x, y)))$

**Exercise 6.3:** (8 P)

Check whether the following first-order formulas are satisfiable, valid or unsatisfiable, where  $a$  and  $b$  are constants and  $g$  is a unary function symbol. Assume a one-sorted signature. Justify your claims by exploring first-order logic semantics.

1.  $(\forall x. \exists y. R(x, y)) \rightarrow R(a, b)$
2.  $(P(a) \wedge \forall x. (P(x) \rightarrow P(g(x)))) \rightarrow P(g(g(a)))$
3.  $(\exists x. P(x)) \rightarrow P(b)$
4.  $P(b) \rightarrow (\exists x. P(x))$

Submit your solution in lecture hall E1.3, Room 001 during the lecture on December 13. Please write your name and the date/time of your tutorial group (Wed-Fabian, Wed-Tobias) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.