max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Christoph Weidenbach **January 6, 2015**

Tutorials for "Automated Reasoning"
Exercise sheet 9

Exercise 9.1: (*4 P*)
Check whether the following first-order formulas are satisfiable, valid or unsatisfiable, where a and b are constants and g is a unary function symbol.

1. $(\forall x.\exists y.R(x,y)) \rightarrow R(a,b)$
2. $(P(a) \wedge \forall x.(P(x) \rightarrow P(g(x)))) \rightarrow P(g(g(a)))$
3. $(\exists x.P(x)) \rightarrow P(b)$
4. $P(b) \rightarrow (\exists x.P(x))$

Exercise 9.2: (*4 P*)
Provide first-order formulas such that the domain of any interpretation satisfying the formula

1. has exactly 3 elements
2. is infinite

Exercise 9.3: (*2 P*)
Let $a :\rightarrow S$ and $R \subseteq S \times T$. Complete the sort information for g, f, P, a and variables x, y such that it enables the following formula: $\forall x, y.(R(x, g(x)) \rightarrow (f(g(x), a) \approx y \vee P(y) \vee R(x, y)))$

Exercise 9.4: (*4 P*)
Prove that the formula $\forall x.((P(x) \leftrightarrow Q(x)) \vee Q(g(x)))$ is satisfiable iff the formula $(\forall x.(R(x) \vee Q(g(x)))) \wedge \forall x.(R(x) \leftrightarrow (P(x) \leftrightarrow Q(x)))$ is satisfiable.

Submit your solution in lecture hall E1.3, Room 002 during the lecture on January 13. Please write your name and the date of your tutorial group (Mon, Thu) on your solution.

Joint solutions are not permitted, please submit individually. However, I encourage you working and solving the exercises in a group.