UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture "Automated Reasoning"
(Winter Term 2014/2015)

Final Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2a	2b	2c	3a	3b	3c	4	5	6a	6b	6c	7	Σ
Answered?														
Points														

Problem 1 *(CDCL)* (5 points)

Check whether the below clause set is satisfiable by running the CDCL calculus to a final state.

1	$P1 \lor P2 \lor P3$
2	$P4 \lor P5 \lor P6$
3	$\neg P3 \lor P5$
4	$\neg P1 \lor \neg P4$
5	$\neg P2 \lor \neg P5$
6	$\neg P3 \lor \neg P6$
7	$\neg P6$
8	$\neg P5 \lor P2$

Problem 2 *(Superposition Model Building)* (6 + 2 + 4 = 12 points)

Consider the below clause set N, $\Sigma = (\{S\}, \{g, b, a\}, \{P, R\})$, with a KBO ordering where all signature symbols and variables have weight 1 and atoms are compared like terms with precedence $P \succ R \succ g \succ b \succ a$.

1	$\neg P(x) \lor P(g(x))$
2	$\neg P(x) \lor R(x, g(x))$
3	$P(a) \lor P(b)$
4	$\neg R(b, g(b)) \lor P(a)$

(a) Compute $\text{ground}(\Sigma, N)_{\neg R(b,g(b)) \lor P(b)}$, i.e., generate all ground instances of N smaller than $\neg R(b, g(b)) \lor P(b)$ and run the partial model operator.

(b) Determine the minimal false ground clause and its productive counterpart and perform the superposition inference step on the respective first-order clauses from N, not on the ground instances.

(c) Can $\text{ground}(\Sigma, N')_{\neg R(b,g(b)) \lor P(b)}$ be extended to a model for N by adding further (arbitrarily chosen) ground atoms? If no, provide an argument why there is always at least one false clause for any extension, if yes provide the complete model and give an argument why it is a model.

Problem 3 *(Unification)* (2 + 2 + 2 = 6 points)

Check whether the below unification problems have a solution using $\Rightarrow_{\mathrm{PU}}$ (polynomial unification). As usual x, y, z, possibly indexed, are variables. If a unifier exists, present it.

(a) $\{f(g(x, y), z) = z_1, z_1 = x_1, x_1 = f(y_1, h(z_1, a))\}$

(b) $\{f(g(x, y), z) = z_1, z_1 = f(y_1, h(x_2, a)), x_2 = g(a, b)\}$

(c) $\{f(z, g(x, y)) = f(x_1, x_1), x = h(y_1, y_1), y = h(z_1, z_1)\}$

Problem 4 *(First-Order CNF)* (6 points)

Transform the formula

$$\neg(\forall x.(\exists y.(R(y,y) \land \forall z.(R(x,z) \lor \neg R(y,x)))))$$

into CNF using algorithm cnf.

Problem 5 *(Tableaux)* (4 points)

Prove that the formula

$$\exists x.(\forall y.((P(x) \to R(x, y)) \vee P(a)))$$

is valid by free-variable tableaux. (Don't forget to negate the formula!)

Problem 6 *(Statements)* (2 + 2 + 2 = 6 points)

Which of the following statements are true or false? Provide a proof or a counter example.

(a) If N is a non-empty and saturated set of first-order clauses then each clause in N has a unique maximal (or selected) literal.

(b) If C_1 and C_2 are two clauses with a common ground instance C, i.e., $C_1\sigma_1 = C$ and $C_2\sigma_2 = C$ for two substiutions σ_1, σ_2 then there is a substitution τ such that $C_1\tau = C_2$ or $C_2\tau = C_1$.

(c) If two terms are comparable with respect to a KBO instance, then they are comparable with respect to an LPO instance.

Problem 7 *(Saturated Clause Sets)* (4 points)

Let N be a finite, satisfiable, saturated first-order Horn clause set. A clause is Horn if it has at most one positive literal. Furthermore, for each Horn clause $(D \lor L) \in N$ with positive literal L, it contains all variables of the clause, i.e., $\text{vars}(D) \subseteq \text{vars}(L)$. Let C be a non-empty ground clause containing only negative literals. Prove that under these assumptions it is decidable whether $N \cup \{C\}$ is unsatisfiable.