Firstly, we define the classic Herbrand interpretations for formulas without equality.

3.5.1 Definition (Herbrand Interpretation)

A *Herbrand Interpretation* (over Σ) is a Σ-algebra $\mathcal{H}$ such that

1. $S^{\mathcal{H}} := T_S(\Sigma)$ for every sort $S \in \mathcal{S}$
2. $f^{\mathcal{H}} : (s_1, \ldots, s_n) \mapsto f(s_1, \ldots, s_n)$ where $f \in \Omega$, arity$(f) = n$, $s_i \in S_i^{\mathcal{H}}$ and $f : S_1 \times \ldots \times S_n \to S$ is the sort declaration for f
3. $P^{\mathcal{H}} \subseteq (S_1^{\mathcal{H}} \times \ldots \times S_m^{\mathcal{H}})$ where $P \in \Pi$, arity$(P) = m$ and $P \subseteq S_1 \times \ldots \times S_m$ is the sort declaration for P

3.5.2 Lemma (Herbrand Interpretations are Well-Defined)

Every Herbrand Interpretation is a Σ-algebra.

3.5.3 Proposition (Representing Herbrand Interpretations)

A Herbrand interpretation $\mathcal{A}$ can be uniquely determined by a set of ground atoms I

$$(s_1, \ldots, s_n) \in P^{\mathcal{A}} \text{ iff } P(s_1, \ldots, s_n) \in I$$

3.5.5 Theorem (Herbrand)

Let N be a finite set of Σ-clauses. Then N is satisfiable iff N has a Herbrand model over Σ iff $\text{ground}(\Sigma, N)$ has a Herbrand model over Σ.