Polynomial Unification

The second calculus, polynomial unification, prevents the problem of exponential growth by introducing an implicit representation for the mgu.

For this calculus the size of a normal form is always polynomial in the size of the input unification problem.



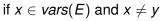
Tautology
$$E \uplus \{t = t\} \Rightarrow_{\mathsf{PU}} E$$

Decomposition
$$E \uplus \{f(s_1, \ldots, s_n) = f(t_1, \ldots, t_n)\} \Rightarrow_{\mathsf{PU}} E \uplus \{s_1 = t_1, \ldots, s_n = t_n\}$$

Clash $E \uplus \{f(t_1, \ldots, t_n) = g(s_1, \ldots, s_m)\} \Rightarrow_{\mathsf{PU}} \bot$ if $f \neq g$



Occurs Check $E \uplus \{x = t\} \Rightarrow_{PU} \bot$ if $x \neq t$ and $x \in vars(t)$ Orient $E \uplus \{t = x\} \Rightarrow_{PU} E \uplus \{x = t\}$ if $t \notin X$ Substitution $E \uplus \{x = y\} \Rightarrow_{PU} E\{x \mapsto y\} \uplus \{x = y\}$





Cycle $E \uplus \{x_1 = t_1, \dots, x_n = t_n\} \Rightarrow_{PU} \bot$ if there are positions p_i with $t_i|_{p_i} = x_{i+1}, t_n|_{p_n} = x_1$ and some $p_i \neq \epsilon$

 $\begin{array}{ll} \text{Merge} & E \uplus \{x=t, x=s\} \Rightarrow_{\mathsf{PU}} E \uplus \{x=t, t=s\} \\ \text{if } t, s \notin \mathcal{X} \text{ and } |t| \leq |s| \end{array}$



3.7.4 Theorem (Soundness, Completeness and Termination of $\Rightarrow_{\mathsf{PU}})$

If s, t are two terms with sort(s) = sort(t) then

- 1. if $\{s = t\} \Rightarrow_{PU}^{*} E$ then any equation $(s' = t') \in E$ is well-sorted, i.e., sort(s') = sort(t').
- 2. $\Rightarrow_{\mathsf{PU}}$ terminates on $\{s = t\}$.
- 3. if $\{s = t\} \Rightarrow_{PU}^{*} E$ then σ is a unifier (mgu) of E iff σ is a unifier (mgu) of $\{s = t\}$.
- 4. if $\{s = t\} \Rightarrow_{PU}^* \bot$ then *s* and *t* are not unifiable.



3.7.5 Theorem (Normal Forms Generated by \Rightarrow_{PU})

Let $\{s = t\} \Rightarrow_{PU}^* \{x_1 = t_1, \dots, x_n = t_n\}$ be a normal form. Then

- 1. $x_i \neq x_j$ for all $i \neq j$ and without loss of generality $x_i \notin vars(t_{i+k})$ for all $i, k, 1 \leq i < n, i+k \leq n$.
- 2. the substitution $\{x_1 \mapsto t_1\}\{x_2 \mapsto t_2\} \dots \{x_n \mapsto t_n\}$ is an mgu of s = t.

