UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture "Automated Reasoning"
(Winter Term 2014/2015)

Midterm Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2a	2b	2c	3	4	5	6a	6b	6c	7	Σ
Answered?												
Points												

Problem 1 *(Superposition)* (8 points)

Show unsatisfiability of the below clause set N via the superposition calculus based on the atom ordering $P_1 \succ P_4 \succ P_5 \succ P_2 \succ P_3$.

1	$P_1 \lor P_2 \lor P_3$	2	$\neg P_1 \lor \neg P_2$	3	$\neg P_2 \lor \neg P_3$
4	$\neg P_1 \lor \neg P_3$	5	$P_4 \lor P_5 \lor P_1$	6	$\neg P_4 \lor P_1$
7	$\neg P_4 \lor P_2$	8	$\neg P_5 \lor P_2$	9	$\neg P_5 \lor P_3$
10	$\neg P_1 \lor P_4$				

Problem 2 *(Superposition Model Building)* (4 + 1 + 2 = 7 points)

Consider again the clause set N of Problem 1, containing the below clauses, but now with *different* atom ordering $P_5 \succ P_4 \succ P_3 \succ P_2 \succ P_1$.

1	$P_1 \vee P_2 \vee P_3$	2	$\neg P_1 \vee \neg P_2$	3	$\neg P_2 \vee \neg P_3$
4	$\neg P_1 \vee \neg P_3$	5	$P_4 \vee P_5 \vee P_1$	6	$\neg P_4 \vee P_1$
7	$\neg P_4 \vee P_2$	8	$\neg P_5 \vee P_2$	9	$\neg P_5 \vee P_3$
10	$\neg P_1 \vee P_4$				

(a) Compute $N_{\mathcal{I}}$.

(b) Determine the minimal false clause and its productive counterpart, producing the atom of the maximal negative literal in the false clause.

(c) Compute the superposition inference out of (b), add it to N resulting in N' and compute $N'_{\mathcal{I}}$.

Problem 3 *(CDCL)* (6 points)

Check via CDCL whether the below clause set is satisfiable.

1	$P_1 \vee P_2 \vee P_3$	2	$\neg P_1 \vee \neg P_2$	3	$\neg P_2 \vee \neg P_3$
4	$\neg P_1 \vee \neg P_3$	5	$P_4 \vee P_5$	6	$\neg P_4 \vee \neg P_1$
7	$\neg P_4 \vee \neg P_2$	8	$\neg P_5 \vee \neg P_3$	9	$\neg P_1 \vee \neg P_5$
10	$\neg P_3 \vee P_5$				

Problem 4 *(CNF)* (6 points)

Transform the formula

$$(P \vee (Q \wedge \neg R)) \vee (P \leftrightarrow (Q \leftrightarrow \bot))$$

into CNF using acnf extended by the elimination rules

ElimTB7 $\chi[\phi \leftrightarrow \top]_p \Rightarrow_{\mathrm{BCNF}} \chi[\phi]_p$

ElimTB8 $\chi[\phi \leftrightarrow \bot]_p \Rightarrow_{\mathrm{BCNF}} \chi[\neg\phi]_p$

Problem 5 *(Tableaux)* (4 points)

Prove that the formula

$$(P \to Q) \to [(R \vee P) \to (R \vee Q)]$$

is valid using tableaux.

Problem 6 *(Superposition Conjectures)* (2 + 2 + 2 = 6 points)

Which of the following statements are true or false? Provide a proof or a counter example.

1. If $N_{\mathcal{I}} \models N$ then N is saturated.
2. If $\delta_C = \{P\}$ while constructing $N_{\mathcal{I}}$ then for all clauses $D = P \lor D'$ with $C \neq D$ we have $\delta_D = \emptyset$, $D \in N$.
3. If all clauses in N have at most one positive literal and there is no clause in N having only negative literals then $N_{\mathcal{I}} \models N$.

Problem 7 *(Polarity Dependent Replacement)* (4 points)

Consider a formula ϕ, position $p \in \text{pos}(\phi)$, $\text{pol}(\phi, p) = 1$ and (partial) valuation $\mathcal{A}$ with $\mathcal{A}(\phi) = 1$. Furthermore, assume that for any position $q < p$ also $\text{pol}(\phi, q) = 1$. Show that if for some arbitrary formula ψ, $\mathcal{A}(\psi) = 1$ then $\mathcal{A}(\phi[\psi]_p) = 1$.